Probabilistic Inference

Probabilistic inference algorithms are targeted towards:
- either continuous distributions: symbolic inference, Hamilton Monte Carlo, variational Bayesian Inference, ...
- or discrete distributions: SAT, weighted model counting, ...

We want to combine state-of-the-art from both → best of both worlds!

Weighted Model Integration

\[
\text{working} \leftarrow (\text{cooling} \land (t^2 < 30)) \lor (t < 5)
\]

\[
p(\text{cooling}) = 0.99 \quad t \sim \mathcal{N}(20, 5)
\]

Question:

\[
p(\text{working}) = ?
\]

In general:

\[
p(x|e) = \frac{p(e|x)p(x)}{\int_x p(x, e)}
\]

Two Algorithms

Exact: Symbo

Approximate: Sampo

\[
t \approx [2.8, 35.1, 5.4, 22.2, 21.4]
\]

\[
\begin{align*}
2 & \quad 2 & \quad 5 & \quad 2 & \quad 5 \\
1 & \quad 1 & \quad 3 & \quad 5 & \quad 2 & \quad 5 \\
[2.5 & \quad 3.5 & \quad 5 & \quad 7.5 & \quad 10 & \quad 15] & \times & (0.99 & \quad 0.99 & \quad 0.99 & \quad 0.99 & \quad 0.99) \\
\end{align*}
\]

\[
\begin{align*}
2 & \quad 2 & \quad 5 & \quad 2 & \quad 5 \\
1 & \quad 1 & \quad 3 & \quad 5 & \quad 2 & \quad 5 \\
[2.5 & \quad 3.5 & \quad 5 & \quad 7.5 & \quad 10 & \quad 15] & \times & (0.99 & \quad 0.99 & \quad 0.99 & \quad 0.99 & \quad 0.99) \\
\end{align*}
\]

\[
[2 & \quad 3 & \quad 5 & \quad 7 & \quad 10 & \quad 21] \\
\]

\[
\begin{bmatrix}
2 & 3 & 5 & 7 & 10 & 21
\end{bmatrix}
\]

\[
p(\text{working}) = \sum_{i=1}^{5} p_{\text{MC, broken}} = 1.99/5 = 0.398
\]

Pure vector calculus and can be executed on the GPU!

\[
\rightarrow \text{cheap probabilistic inference} \\
\rightarrow \text{embarrassingly parallelizable}
\]

Integrals become easily intractable.

Knowledge Compilation

offline: compile theory (expensive)

online: fast inference (cheap)

- evaluation in linear time
- conditioning in poly-time
- repeated querying

\[
\text{working} \leftrightarrow \text{cooling} \lor \text{low}_t
\]

Algebraic Model Counting

Generalized framework for probabilistic inference:

- define specific semiring \((A, \oplus, \otimes, e^\oplus, e^\otimes)\) for specific task

Link to belief propagation:

- sum-product: \(\oplus\) is normal addition
- max-product: \(\oplus\) is maximization

We defined a custom probability density semiring with custom elements:

\[
A = \{ (a, \mathcal{V}(a)) \}
\]

\[
a = [t^2 < 30] + 0.99[t^2 < 30][t < 5] \\
V(a) = \{ t \}
\]

Results

- Symbo is faster on 9/10 benchmark problems than PSI, excluding knowledge compilation
- Symbo is faster on 7/10 benchmark problems than PSI, including knowledge compilation

Logical reasoning generally improves symbolic inference!

- Sampling on the GPU \(\rightarrow\) constant time complexity
- Avoid sampling categorical variables \(\rightarrow\) reduction in variance

Adnan Darwiche. "Modeling and reasoning with Bayesian networks".  
Angelika Kimmig, Guy Van den Broeck, and Luc De Raedt. "Probabilistic logic programming for hybrid relational domains".  
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Brian Milik, Blaauw Martijn, and Stuart Russell. "BLOG: Relational modeling with unknown objects".